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Corrigendum

Corrigendum to "Unconditional martingale difference sequences in Banach spaces" [J. Math. Anal. Appl. 326 (2007) 1291–1307]

Stuart F. Cullender *, Coenraad C.A. Labuschagne

School of Mathematics, University of the Witwatersrand, Private Bag 3, PO WITS 2050, South Africa

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We regret to announce that there is a mathematical error on page 1300, which adversely affects the Abstract, Theorems 5.2 and 7.2, and Corollaries 7.3 and 7.4.

We recall that if X and Y are Banach spaces, a sequence $(x_i) \subset X$ is said to be *equivalent* to a sequence $(y_i) \subset Y$ provided, for all scalar sequences (α_i), we have $\sum_{i=1}^{\infty} \alpha_i x_i$ convergent $\Leftrightarrow \sum_{i=1}^{\infty} \alpha_i y_i$ convergent. If $(x_i) \subset X$ is a basic sequence that is equivalent to a sequence $(y_i) \subset Y$, then there exists a constant K such that $\|\sum_{i=1}^{n} \alpha_i y_i\| \leqslant K \|\sum_{i=1}^{n} \alpha_i x_i\|$ for any choice of scalars $\alpha_1, \ldots, \alpha_n$ and $n \in \mathbb{N}$ (cf. [3, Chapter I, Theorem 8.1]).

We noted on page 1300 that the following inequality holds: For any unconditional basic sequence $(x_n) \subset L^p(\mu)$ with unconditional constant M, where $1 \leqslant p < \infty$, there exist constants K_1 and K_p (K_p dependent on p) for which

$$K_1 (M K_p)^{-1} \left\| \sum_{i=1}^{n} \alpha_i x_i \right\|_{L^p(\mu)} \leqslant \int_0^1 \left\| \sum_{i=1}^{n} r_i(t) \alpha_i |x_i| \right\|_{L^p(\mu)} \mathrm{d}t \leqslant K_1^{-1} (M K_p) \left\| \sum_{i=1}^{n} \alpha_i x_i \right\|_{L^p(\mu)}$$

holds for all scalars $\alpha_1, \ldots, \alpha_n$ (where (r_i) denotes the sequence of Rademacher functions). Using the unconditionality of (x_i) and the above inequality, we concluded (incorrectly) that (x_i) is equivalent to $(|x_i|)$. This conclusion arose from our erroneous interpretation of the word 'equivalent' used in [1, p. 128]. Our conclusion is easily seen to be true when (x_i) is a mutually disjoint sequence. But, in general, this is hardly the case, as the following simple counterexample shows.

Let $(r_i) \subset L^2(\mu)$ denote the sequence of Rademacher functions. Since (r_i) is equivalent to the unit vector basis of ℓ^2, it follows that (r_i) is an unconditional basic sequence. If our claim were true, then there exists a constant $K > 0$ such that

$$\left\| \sum_{i=1}^{n} \alpha_i |r_i| \right\|_2 \leqslant K \left\| \sum_{i=1}^{n} \alpha_i r_i \right\|_2$$

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* Corresponding author.
E-mail address: scullender@gmail.com (S.F. Cullender).

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for any choice of scalars $\alpha_1, \dots, \alpha_n$ and $n \in \mathbb{N}$. But then

$$n = \left\| \sum_{i=1}^{n} \mathbf{1} \right\|_2 = \left\| \sum_{i=1}^{n} |r_i| \right\|_2 \leqslant K \left\| \sum_{i=1}^{n} r_i \right\|_2 = K \left(\sum_{i=1}^{n} \int_0^1 r_i^2(t)\,\mathrm{d}t \right)^{1/2} = K n^{1/2}$$

for all $n \in \mathbb{N}$, which is absurd.

The affected results mentioned above still hold for disjoint sequences. However, corrected results for general sequences may be found in [2].

References

[1] D. Alspach, E. Odell, L_p spaces, in: Handbook of the Geometry of Banach Spaces, vol. 1, North-Holland, Amsterdam, 2001, pp. 123–159.
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[3] I. Singer, Bases in Banach Spaces I, Grundlehren Math. Wiss., vol. 154, Springer-Verlag, 1970.